

Fluids Quiz 2 Solution

Fall '07

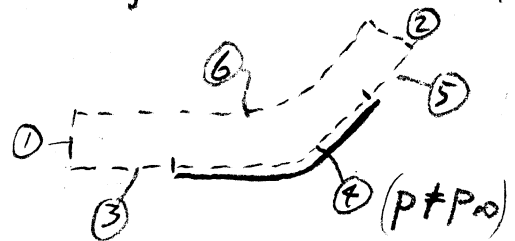
1) Alternative CV (more difficult)

a)



$$b) \oint \rho \vec{v} \cdot \hat{n} dA = 0 \quad \text{same as before}$$

$$c) \oint \rho (\vec{v} \cdot \hat{n}) \vec{v} dA = \rho V^2 A \left[\left(\frac{1}{\sqrt{2}} - 1 \right) \hat{i} + \frac{1}{\sqrt{2}} \hat{j} \right] \quad \text{same as before}$$

d) Note: Now, $p \neq p_\infty$ over deflector

$$\oint \rho \hat{n} dA = \int_1 p_\infty \hat{n} dA + \int_2 p_\infty \hat{n} dA + \int_3 p_\infty \hat{n} dA + \int_4 p \hat{n} dA + \int_5 p_\infty \hat{n} dA + \int_6 p_\infty \hat{n} dA$$

Add and subtract $\int_4 p_\infty \hat{n} dA \dots$

$$\boxed{\oint \rho \hat{n} dA = \cancel{\oint p_\infty \hat{n} dA} + \int_4 (\rho - p_\infty) \hat{n} dA = \int_4 (\rho - p_\infty) \hat{n} dA}$$

$$e) \text{ We also note that } \int_4 (\rho - p_\infty) \hat{n} dA = \vec{R} \rightsquigarrow \begin{array}{c} \text{Diagram showing pressure forces on the deflector: } p \hat{n} \text{ (upward) and } -p_\infty \hat{n} \text{ (downward).} \\ \text{integrated pressure forces} \end{array}$$

Integral momentum eq'n: $\oint \rho (\vec{v} \cdot \hat{n}) \vec{v} + \oint p \hat{n} dA = 0$ (nothing in side)

$$\therefore \oint p \hat{n} dA = - \oint \rho (\vec{v} \cdot \hat{n}) \vec{v} dA$$

$$\text{or } \int_4 (\rho - p_\infty) dA = \vec{R} = \rho V^2 A \left[\left(1 - \frac{1}{\sqrt{2}} \right) \hat{i} - \frac{1}{\sqrt{2}} \hat{j} \right] \quad \text{same as before}$$